

Points of Concurrency of a Triangle: the incenter, the circumcenter, the centroid and the orthocenter

The incenter

1. With the straightedge tool, draw a triangle out of three line segments, and label the vertices *A*, *B* and *C*.
2. Construct the angle bisectors for the triangle
 - a. Click on some blank space, so that you're not selecting anything.
 - b. Click on the three triangle vertices, one after the other.
 - c. Select Construct/Angle Bisector. A ray will be constructed that bisects the middle point that you selected.
 - d. Repeat the process two more times so that the other angle bisectors are constructed.
3. Notice that the three angle bisectors all intersect at the same point. Using the Selection Arrow tool, click on the intersection. A point will be highlighted at that intersection. This is the ***incenter*** of the triangle. Label it *I*.
4. Select the three angle bisectors (and nothing else). In the Edit menu, go to Action Buttons, and select Hide/Show. A button will appear that says Hide Bisectors. If you click it, the bisectors will disappear, and the button will now read Show Bisectors. Click it on and off to make the bisectors appear and reappear. Once you see how it works, leave them hidden.
5. Right-click the button, and change the label to read "Show Angle Bisectors" instead of "Show Bisectors". This will help clarify things later.
6. Construct a line through the incenter that is perpendicular to one of the edges of the triangle.
 - a. Select the incenter and one of the edges of the triangle (and nothing else).
 - b. Select Construct/Perpendicular Line.
7. Inscribe a circle in the triangle, with the incenter as the center of the circle.
 - a. With the circle tool, click on the incenter.
 - b. Move the mouse to the intersection of the perpendicular line that you just constructed, and the edge that forms a right angle with it (when both lines are highlighted, you're there). Click on the intersection to draw the circle.
8. Select the perpendicular line that you constructed, and the intersection point that you used to draw the circle. Don't select anything else.
9. Type Ctrl-H (or select Display/Hide Objects) to hide the line and point. We're not going to worry about making it reappear, so we won't make a button to hide it.
10. Wiggle the vertices around, and notice that the circle is always perfectly inscribed in the triangle.
11. Select only the circle, and make a Hide/Show button for the circle just like the last Hide/Show button you made. Relabel the button "Show Inscribed Circle" instead of just "Show Circle". Leave the circle hidden.

The circumcenter

12. Now select the three edges of the triangle (and nothing else).
13. Choose Construct/Midpoints (or type Ctrl-M) to construct midpoints of the segments. Make another Hide/Show button for the midpoints. Don't relabel it, since it's already pretty well labeled. Leave the midpoints showing.
14. Draw perpendicular bisectors of the edges of the triangle
 - a. Select an edge and its midpoint (and nothing else).
 - b. Select Construct/Perpendicular Line.
 - c. Repeat for the other two edges of the triangle.

15. Using the Selection Arrow tool, click on the intersection of the three perpendicular lines that you just constructed. A point will be highlighted at that intersection. This is the *circumcenter* of the triangle. Label it ***Cr***.
16. Make another Hide/Show button that hides and shows the perpendicular bisectors. Change its label from “Show Perpendicular Lines” to “Show Perpendicular Bisectors”.
17. Now with the circle tool, circumscribe the triangle.
 - a. With the circle tool, click on the circumcenter.
 - b. Click on any of the triangle’s vertices to draw the circle.
18. Wiggle the vertices of the triangle around to verify that the circle is always perfectly circumscribed about the triangle.
19. With the buttons you made, show both the angle bisectors and the perpendicular bisectors at the same time. Other than the vertices of the triangle, what other points do you notice that lie on this circle?
20. Select only the circle, and make a Hide/Show button for the circle. Relabel the button “Show Circumscribed Circle” instead of just “Show Circle”. Leave the circle hidden.
21. Click your Show/Hide buttons so that only the triangle, ***Cr*** and ***I*** are showing.

The centroid

22. Click on the “Show Midpoints” button. Now draw the three medians for the triangle, by making a segment from each vertex to its corresponding midpoint.
23. Using the Selection Arrow tool, click on the intersection of the three medians that you just constructed. A point will be highlighted at that intersection. This is the *centroid* of the triangle. Label it ***Cn***.
24. Make a Hide/Show button for the medians as well. Relabel it from “Show Segments” to “Show Medians”.
25. Hide the medians and the midpoints, so that only the triangle, the incenter, the circumcenter and the centroid are showing.

The orthocenter

26. Construct the three altitudes of the triangle.
 - a. Select one of the vertices and the opposite side of the triangle. Choose Construct/Perpendicular Line.
 - b. Do the same for the other two vertices of the triangle.
27. Using the Selection Arrow tool, click on the intersection of the three altitudes that you just constructed. A point will be highlighted at that intersection. This is the *orthocenter* of the triangle. Label it ***O***. Also click on each intersection between a side of the triangle and an altitude. (Your triangle will need to be acute for you to get all three intersections.)
28. Make another Show/Hide for the altitudes and their intersections with the opposite sides. Relabel this button “Show Altitudes”.
29. Points ***I***, ***Cr***, ***Cn*** and ***O*** are called points of concurrency of the triangle. Move the vertices so that all four points of concurrency lie on top of each other. What type of triangle do you see?
30. Wiggle the vertices so that the points don’t all line up. Draw a segment from one point of concurrency to another so that another one of the points is on the segment. This segment is called the “Euler segment” of the triangle. Which are the two endpoints of the Euler segment? Which point does the segment pass through?

31. Move the vertices so that all four points of concurrency are lined up (that is, that the fourth point also lies on the segment). What type of triangle do you see now?
32. Move the vertices so that not all of the points of concurrency are inside the triangle. What type of triangle do you see now? Which points lie outside the triangle? Which points are always in the triangle?
33. Move the vertices so that one of the points of concurrency lies on top of one of the vertices. Which point can you get to lie on top of a vertex? What type of triangle do you see now? Is another point lying anywhere significant?
34. Move the vertices around again so that no points are lying on top of each other. Measure the distances along the two parts of the Euler segment.
 - a. Select one endpoint of the Euler segment, and the point that lies on the segment. Select Measure/Distance.
 - b. Do the same for the other endpoint of the Euler segment.
 - c. How do these distances compare with each other?
35. Construct the midpoint of the Euler segment.
 - a. Select the segment, and select Construct/Midpoint.
 - b. Label the midpoint E .
36. Draw a circle that is centered at point E , and which passes through the midpoint of one of the sides. This is called the “nine-point circle”, because it passes through nine significant points. What are six of those points?
37. Draw segments that connect the orthocenter O with the three vertices of the triangle. Construct the midpoints of those segments. Now you should see the nine points that give the circle its name.