

Drawing a Hyperbola in Sketchpad

1. Plot two points on the page, and label them $F[1]$ and $F[2]$. When displayed, the labels will look like F_1 and F_2 .
2. Draw a circle that is centered at F_1 . Make sure that F_2 is not in the circle. Put point P on the circle. When you wiggle point P around, it should stay on the circle, and should not move the circle or change its size.
3. Draw a segment from P to F_2 .
4. Construct the midpoint of segment $\overline{F_2P}$. Label it G .
5. Draw a line through P and F_1 .
6. Construct a line perpendicular to $\overline{F_2P}$ that goes through G .
7. Label the intersection of this new perpendicular line and $\overline{F_1P}$. Call it E .
8. Draw a segment from E to F_2 .
9. Note that $\triangle EGP$ and $\triangle EGF_2$ are congruent right triangles. In particular, notice that $EP = EF_2$, no matter what the location of P .
10. Move P around the circle. Notice that sometimes P is between F_1 and E , and that sometimes F_1 is between P and E .

When P is between F_1 and E , then $EF_1 - EF_2 = EF_1 - EP = PF_1$, which is constant, since it's the radius of a circle. So $EF_1 - EF_2$ is constant.

When F_1 is between P and E , then $EF_2 - EF_1 = EP - EF_1 = PF_1$. Again, this is the same constant.

This means that as you move P around the circle and E moves all over the place, $|EF_1 - EF_2| = PF_1$, which is a constant.

11. Select point E and display the traces. Now when you move P around the circle, you should recognize the shape that E traces.
12. For a cool effect, trace the line \overline{EG} .
13. Play around with it a little, moving the circle closer to F_2 and then farther away. Remember that you can erase your traces by typing Ctrl-B. When is the shape narrower? When is it wider?
14. Now before you declare this to be the coolest thing in the world, make the circle wider so that EF_2 is inside the circle. Now move point P around again, and see what shape you're drawing. What happened? Why did it happen?