Multiplying Complex Numbers With Geometer's Sketchpad.

1. Start a new sketch; show the graph.
2. Give point (0,0) the label *O* and point (1,0) the label *P*.
3. Click on Graph/Snap Points.
4. Put 2 points anywhere on the graph (other than the axes). Label these points *A* and *B*. Wiggle them around and notice that they will stick onto points with integer coefficients.
5. Draw segements from *O* to each of points *A* and *B*.
6. Select points *A* and *B* and click on Measure/Coordinates.

Note: Points *A* and *B* represent two complex numbers. If point A has the coordinates (2, 3), then it represents the number $2+3i$. We're going to find out geometrically what the product of these two numbers is.

7. Draw a circle of radius 1 around the origin. Do this by selecting the circle tool, and then clicking down at point O and releasing at point P. After drawing the circle, move point P left and right (thereby zooming in and out of your grid) and make sure the circle moves with it.
8. Click on the intersection of your segments and the circle. Two points should appear.
9. Now click (in precise order) on the points, P, O, and the intersection of the circle and $\overline{OA}$. Select Mark Angle from the Transform menu. If it's not available, then you probably have too many things selected. Try again with only those points selected, and in the appropriate order.
10. Double click point *O*. A funky bullseye will appear for a second and then go away. You have just marked the origin as the center of all rotations and dilations.
11. Now select the intersection of the circle and $\overline{OB}$, and select Rotate from the Transform menu. A dialogue box will appear, with "Marked Angle" selected. Leave that the way it is, and select "Rotate". Another point will appear.
12. Draw a ray (not a segment or line, but a ray) from *O* to this new point on the circle. Move points A and B around the graph, and notice that the ray moves as well. Also notice that the ray represents the terminal side of the sum of $\measuredangle POA$ and $\measuredangle POB$.
13. Now measure the lengths of segments $\overline{OA}$ and $\overline{OB}$. Do this by selecting points *O* and *A* and then selecting Coordinate Distance from the Measure menu. Do the same with points *O* and *B*. The lengths of the segments should appear right below the coordinates in the upper left part of the screen.
14. Now measure the product of these distances. Do this by clicking Measure/Calculate. A dialogue box appears that looks like a calculator. Click on the measure of $\overline{OA}$ (that you had calculated earlier). When you mouse over it, a light blue box should appear around it. After selecting that, select "*" (the multiplication key), and then the measure of $\overline{OB}$. The dialogue box should read: OA*OB. Click OK, and the product should appear right below the two measurements that it is multiplying.

Note: Remember that any complex number can be expressed either as $x + yi$ or as $r(\cos x + i\sin y)$. We will use the polar form to find the product of the numbers. Recall that
$(x_1 + y_1 i)(x_2 + y_2 i) = r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$.

So in order to find the product, we must find the product of the two segment lengths (which we did in step 13) and the sum of the two angles (which we did in step 11).

15. So in order to find the point that represents the product of *A* and *B*, we have to put a point on the ray that we constructed that is the distance OA*OB from the origin. No problem: Select the intersection of the ray we constructed and the unit circle. Now select Dilate from the Transform Menu. When the dialogue box appears, select "Marked Ratio" and then click on the measurement for OA*OB and then on "Dilate". A point will appear on the ray. If you don't see a point, then move both *A* and *B* closer to the origin and see if you can see the point on the ray.
16. Label this point *C* and measure its coordinates. Note that the coordinates

Using this program you just made, calculate the following:

1. $(4-3i)(6+5i) =$ ____________________
2. $(-3+2i)(2-i) =$ ____________________
3. $(3-6i)(1+2i) =$ ____________________
4. $(6-i)(12+2i) =$ ____________________
5. $(-i)(9-4i) =$ ____________________